M6a: Open Channel Flow
(Manning's Equation, Partially Flowing Pipes, and Specific Energy)

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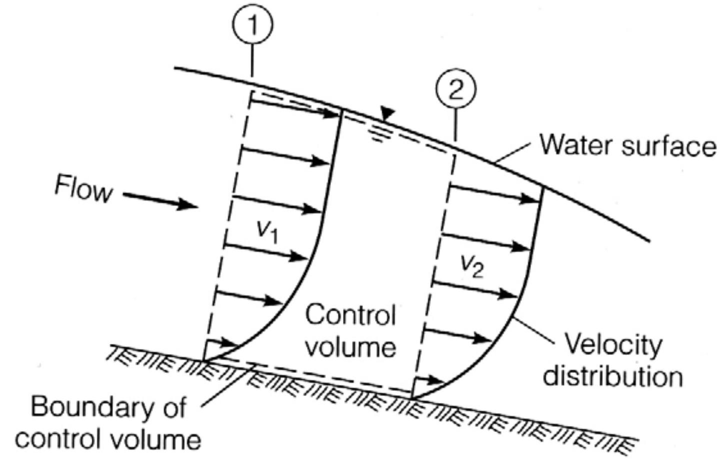
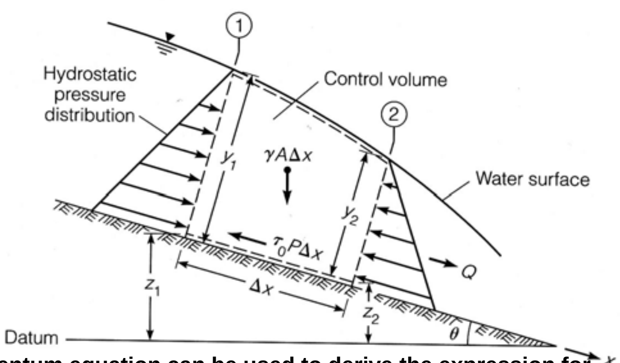
Steady Non-Uniform Flow in an Open Channel



Continuity Equation: $V_1A_1 = V_2A_2$

Chin 2006; Figure 3.1

Steady, Non-Uniform Flow in an Open Channel



The momentum equation can be used to derive the expression for shear stress: $\tau_o = \gamma R S_f$

γ = specific weight of water (62.4 lbs/ft3)
R = hydraulic radius (ft)
Sf = hydraulic slope (ft/ft) (slope of the energy grade line, or the friction slope)

Chin 2006; Figure 3.2

Manning Coefficients for Open Channels (Table 4.1, Chin 2000)

SI units (m/s; m)

$$V = \frac{1}{n} R^{2/3} S_f^{1/2}$$

U.S. Customary units (ft/sec; ft)

$$V = \frac{1.486}{n} R^{2/3} S_f^{1/2}$$

Where Sf is the slope of the energy grade line (friction slope). If the channel slope is used, then implies uniform flow, which is rare.

Material	n
Lined channels:	
Asphalt	0.013–0.017
Brick	0.012–0.018
Concrete	0.011–0.020
Rubble or riprap	0.020–0.035
Vegetal	0.030–0.40
Excavated or dredged channels:	
Earth, straight and uniform	0.020–0.030
Earth, winding, fairly uniform	0.025–0.040
Rock	0.030–0.045
Unmaintained	0.050–0.14
Natural channels*:	
Fairly regular section	0.03–0.07
Irregular section with pools	0.04–0.10

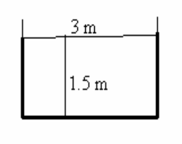
Source: ASCE (1982). Reproduced by permission of the American Society of Civil Engineers.
*Minor streams, top width at flood stage less than 31 m.

Only valid for hydraulically rough flow, when: $n^6 \sqrt{RS_f} \geq 1.9x10^{-13}$ **(SI units)**

Manning’s Equation

Example:

- Determine the flow rate in a rectangular concrete channel with a width of 3 m and a HGL slope of 0.001 m/m when the depth of flow is 1.5 m. Assume n = 0.014.



Given: n = 0.014 (concrete channel)
L = 3 m (width of channel)
w = 1.5 m (depth of flow)
s_f = 0.001 m/m

Manning’s Equation

- Use the Manning's equation:

$$Q = \frac{A}{n} R^{2/3} S_f^{1/2}$$

Need A (cross-sectional area of flow):
A = Lw
Substituting:
A = (3 m)(1.5 m)
A = 4.5 m^2

Manning’s Equation

- Need R (hydraulic radius):
  R = A/P
- Need P, the wetted perimeter (noted on drawing by thicker lines).
  P = L + 2w
  Substituting:
  P = (3 m) + 2(1.5 m)
  P = 6 m

Substituting into equation for hydraulic radius:
R = A/P
R = (4.5 m^2)/(6 m)
R = 0.75 m

Manning’s Equation

- Substituting into Manning's equation:

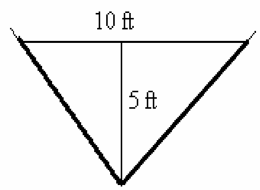
$$Q = \frac{(4.5m^2)(0.75m)^{2/3}(0.001m/m)^{1/2}}{0.014}$$

$$Q = 8.39 m^3/\sec$$

Manning's Equation

Example:

- Given a V-shaped channel with a HGL slope of 0.001, a top width of 10 feet, and a depth of 5 feet, determine the velocity of flow using the Manning's equation. Find the discharge in both ft^3/sec (cfs) and m^3/sec (cms).



Manning's Equation

Substituting:

R = (25 ft^2)/14.14 ft

R = 1.77 ft

Assume that the channel is concrete-lined with a Manning's n of 0.015.

$$Q = \frac{1.49}{n} A R^{2/3} S^{1/2}$$

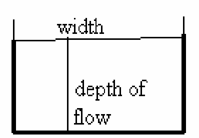
$$Q = \frac{1.49}{0.015}(25\,ft^2)(1.77\,ft)^{2/3}(0.001)^{1/2}$$

Q = 114.9 cfs = 115 ft^3/sec(0.3048 m/ft)^3 = 32.5 m^3/sec

Manning's Equation

Example:

- Find the dimensions of a rectangular concrete channel to carry a flow of 150 m^3/sec, with a HGL slope of 0.015 and a mean velocity of 10.2 m/sec.



Given:

Q = 150 m^3/sec

V = 10.2 m/sec

S_f = 0.015

Assume: Manning's n = 0.013 (concrete channel)

Manning's Equation

- Have everything needed to solve Manning's equation for the hydraulic radius, R:

$$V = \frac{1}{n} R^{2/3} S^{1/2}$$

$$R^{2/3} = \frac{Vn}{S^{1/2}}$$

$$R = \left(\frac{Vn}{S^{1/2}}\right)^{3/2}$$

$$R = \left(\frac{(10.2m/\sec)(0.013)}{(0.015)^{1/2}}\right)^{3/2}$$

$$R = 1.13m$$

Manning's Equation

By definition, R = cross-sectional area of flow/wetted perimeter

$$A = (\text{Base of channel})(\text{Depth of flow})$$
$$P = (\text{Base of channel}) + 2(\text{Depth of flow})$$
$$\text{Substituting:}$$
$$R = \frac{(\text{Base})(\text{Depth})}{[\text{Base} + 2\ \text{Depth}]} = 1.13\text{m}$$

By the Continuity Equation:

$$A = \frac{Q}{V} = (Base)(Depth)$$
$$A = \frac{150 m^3/\sec}{10.2 m/\sec}$$
$$A = 14.7 m^2 = (Base)(Depth)$$

Manning's Equation

- Have two equations and two unknowns:

$$14.7 m^2 = (Base)(Depth)$$
$$Base = \frac{14.7}{Depth}$$
$$1.13 m = \frac{14.7 m^2}{Base + 2Depth}$$
$$Base + 2Depth = 13.0 m$$
$$\left(\frac{14.7}{Depth}\right) + 2Depth = 13.0$$
$$14.7 + 2Depth^2 = 13Depth$$
$$2Depth^2 - 13Depth + 14.7 = 0$$

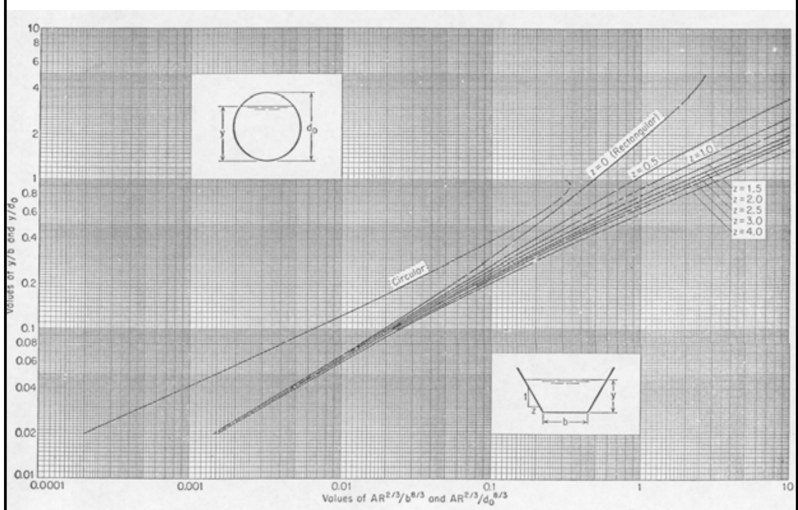
Two possible solutions to quadratic (both are correct):

Base = 2.9 m	Depth of Flow = 5.04 m
Base = 10.1 m	Depth of Flow = 1.46 m

Figure 5-8 (Chow 1959) can be used to significantly shorten the calculation effort for the design of channels. This figure is used to calculate the normal depth (y) of a channel based on the channel side slopes and known flow and channel characteristics, using the Manning's equation in the following form:

$$AR^{\frac{2}{3}} = \frac{nQ}{1.49\, S^{0.5}}$$

Initial channel characteristics that must be know include: z (the side slope), and b (the channel bottom width, assuming a trapezoid). It is easy to examine several different channel options (z and b) by calculating the normal depth (y) for a given peak discharge rate, channel slope, and roughness. The most practical channel can then be selected from the alternatives.



Composite Manning's n Estimate, Example 3.2 (Chin 2006)

A floodplain (next slide) can be divided into seven sections as shown below. Use the various formula in the table to estimate the composite roughness value for this channel.

Section	n
1	0.040
2	0.030
3	0.015
4	0.013
5	0.017
6	0.035
7	0.060

Formula	Reference
$n_e = \left(\frac{\sum_{i=1}^{N} P_i n_i^{3/2}}{\sum_{i=1}^{N} P_i} \right)^{2/3}$	Horton (1933), Einstein (1934)*
$n_e = \frac{\left(\sum_{i=1}^{N} P_i n_i^2 \right)^{1/2}}{\left(\sum_{i=1}^{N} P_i \right)^{1/2}}$	Muhlhofer (1933), Einstein and Banks (1951)
$n_e = \frac{P R^{5/3}}{\sum_{i=1}^{n} \frac{P_i R_i^{5/3}}{n_i}}$	Lotter (1933)†
$\ln n_e = \frac{\sum_{i=1}^{N} P_i y_i^{3/2} \ln n_i}{\sum_{i=1}^{N} P_i y_i^{3/2}}$	Krishnamurthy and Christensen (1972)‡

*Formula assumes that the mean flow in each of the subareas is equal to the mean flow velocity.
† P and R are the perimeter and hydraulic radius of the entire cross-section, respectively.
‡ y_i is the average flow depth in Section i.

Floodplain showing seven separate sections corresponding to different values of n.

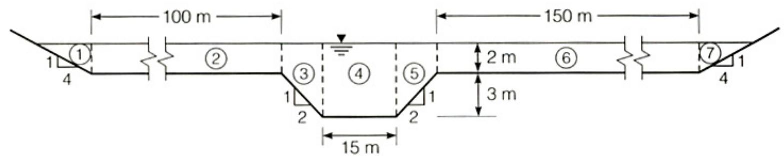


Figure 3.5, Chin 2006

Each section has the following geometric characteristics:

Section, i	P_i (m)	A_i (m^2)	R_i (m)	n_i	y_i (m)
1	8.25	8.00	0.97	0.040	1.00
2	100	200	2.00	0.030	2.00
3	6.71	21	3.13	0.015	3.50
4	15.0	75	5.00	0.013	5.00
5	6.71	21	3.13	0.017	3.50
6	150	300	2.00	0.035	2.00
7	8.25	8.00	0.97	0.060	1.00
	295	633			

These values are used with the prior equations to result in the following estimates for Manning's n:

Formula	n_e
Horton/Einstein	0.033
Muhlhofer/Einstein and Banks	0.033
Lotter	0.022
Krishnamurthy and Christensen	0.026

The estimates of the composite n values can vary considerably, resulting in similar differences in predicted discharges.

As shown earlier, the Manning's equation can also be also used to predict flows in pipes. Drainage systems are typically designed as open channel flows in circular pipes, although other cross-sectional shapes are used.

Charts or tables can be used to help predict the flow conditions in these systems when the pipes are not flowing full.

Sewers Flowing Partly Full

From: Metcalf and Eddy, Inc. and George Tchobanoglous. *Wastewater Engineering: Collection and Pumping of Wastewater*. McGraw-Hill, Inc. 1981.

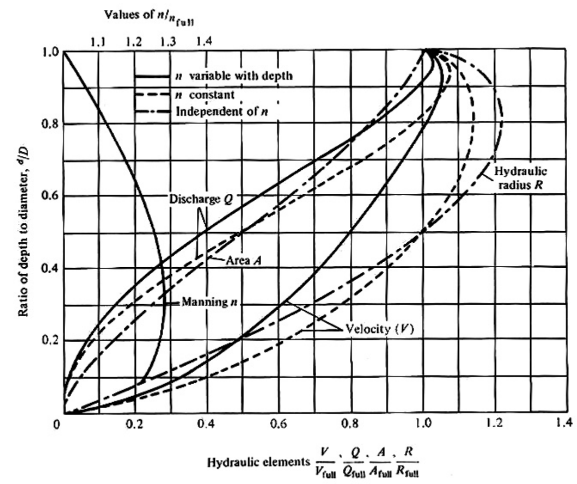


Figure 2-16 Hydraulic elements for circular sewers [10].

Sewers Flowing Partly Full

Table 2-5 Values of K' for circular channels in terms of diameter in the equation[a] $Q = (K'/n)D^{8/3}S^{1/2}$

d[b]/D	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	....	0.000047	0.00021	0.00050	0.00093	0.00150	0.00221	0.00306	0.00407	0.00521
0.1	0.00651	0.00795	0.00953	0.0113	0.0131	0.0152	0.0173	0.0196	0.0220	0.0246
0.2	0.0273	0.0301	0.0331	0.0362	0.0394	0.0427	0.0461	0.0497	0.0534	0.0572
0.3	0.0610	0.0650	0.0691	0.0733	0.0776	0.0820	0.0864	0.0910	0.0956	0.1003
0.4	0.1050	0.1099	0.1148	0.1197	0.1248	0.1298	0.1349	0.1401	0.1453	0.1506
0.5	0.156	0.161	0.166	0.172	0.177	0.183	0.188	0.193	0.199	0.204
0.6	0.209	0.215	0.220	0.225	0.231	0.236	0.241	0.246	0.251	0.256
0.7	0.261	0.266	0.271	0.275	0.280	0.284	0.289	0.293	0.297	0.301
0.8	0.305	0.308	0.312	0.315	0.318	0.321	0.324	0.326	0.329	0.331
0.9	0.332	0.334	0.335	0.335	0.335	0.335	0.334	0.332	0.329	0.325
1.0	0.312									

[a] Adapted from Ref. 2.
where Q = flowrate, m^3/s
n = Manning coefficient of friction
D = diameter of conduit
S = slope of energy grade line, m/m.
[b] d = depth of flow
Note: $m^3/s \times 35.3147 = ft^3/s$
m × 3.2808 = ft

From: Metcalf and Eddy, Inc. and George Tchobanoglous. *Wastewater Engineering: Collection and Pumping of Wastewater*. McGraw-Hill, Inc. 1981.

Sewers Flowing Partly Full

Example:

- Determine the depth of flow and velocity in a sewer with a diameter of 300 mm having a HGL slope of 0.005 m/m with an n value of 0.015 when discharging 0.01 m^3/sec.

Given: D = 300 mm
S_f = 0.005 m/m
n = 0.015
Q = 0.01 m^3/sec

Sewers Flowing Partly Full

- Use the modified Manning's equation for partly full sewers:

$$Q = \left(\frac{K'}{n}\right) D^{8/3} S^{1/2}$$

$$\text{Rearranging}:$$

$$K' = \frac{nQ}{D^{8/3} S^{1/2}}$$

$$\textit{Substituting}:$$

$$K' = \frac{(0.015)(0.01 m^3/\sec)}{(0.3m)^{8/3}(0.005 m/m)^{1/2}}$$

$$K' = 0.0526$$

Sewers Flowing Partly Full

- Using Table 2-5 (equation in terms of diameter of pipe):

  Close to K′ = 0.0534

  Therefore, d/D = 0.28

- Substituting:

  d/(0.3 m) = 0.28

  Depth of flow, d = 0.084 m

Sewers Flowing Partly Full

- To calculate velocity at depth of water of 84 mm, need to use continuity equation:

  Q = VA

- Using Manning's partial flow diagram (assuming a constant n):

  At d/D = 0.28

Sewers Flowing Partly Full

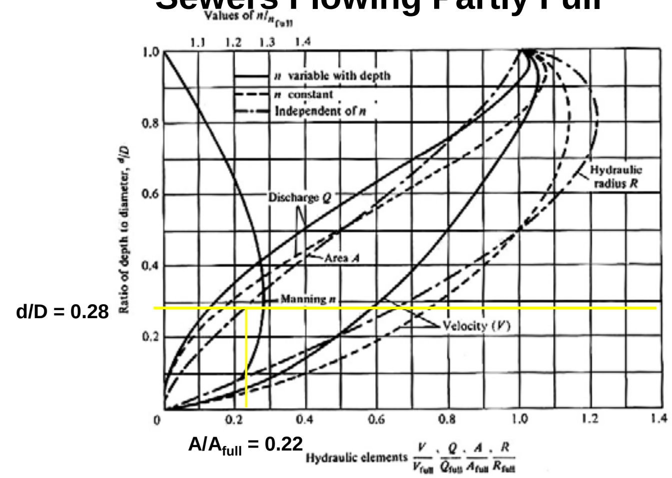


Figure 2-16 Hydraulic elements for circular sewers [10].

From: Metcalf and Eddy, Inc. and George Tchobanoglous. *Wastewater Engineering: Collection and Pumping of Wastewater*. McGraw-Hill, Inc. 1981.

Sewers Flowing Partly Full

- Using Manning's partial flow diagram (assuming a constant n):

  At d/D = 0.28, A/A_{full} = 0.22

- Calculate A_{full}.

$$A_{full} = \left(\frac{\pi}{4}\right)D^2 = \left(\frac{\pi}{4}\right)(0.3m)^2$$
$$A_{full} = 0.0707m^2$$

Sewers Flowing Partly Full

- Substituting:

$$\frac{A}{A_{full}} = 0.22 = \frac{A}{0.0707m^2}$$
$$A = 0.0156m^2$$

- Substituting into the continuity equation:

$$Q = VA$$
$$0.01m^3/\sec = V(0.0156m^2)$$
$$V = 0.641m/\sec$$

In-Class Problem (Partially Flowing Sewer)

- Determine the depth of flow and velocity in a sewer with a diameter of 600 mm having a HGL slope of 0.005 m/m with an n value of 0.013 when discharging 0.055 m^3/sec.

Remember the problem having two “correct” answers:

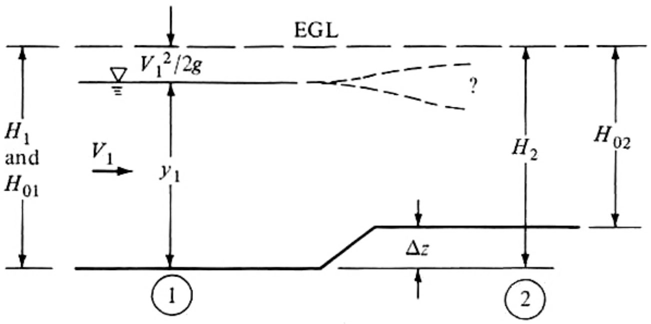


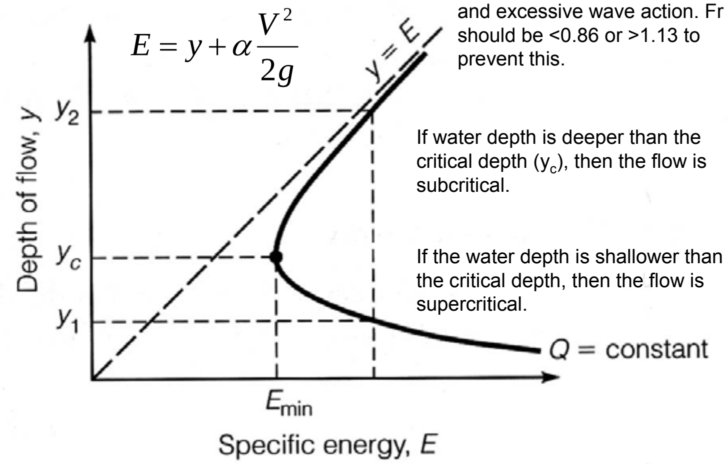
Figure 7–7 Transition in an open channel.

The specific energy diagram is used to determine the most likely water depth.

Prasuhn 1987

Typical Specific Energy Diagram (Figure 3.6, Chin 2006)

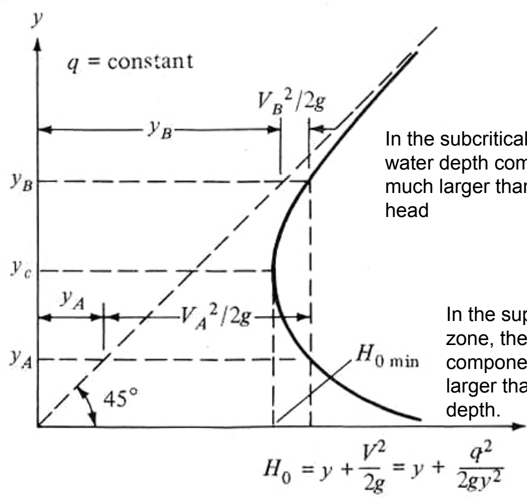
$$E = y + \alpha \frac{V^2}{2g}$$



When the water depth is close to critical, small changes in specific energy results in large depth changes, resulting in unstable and excessive wave action. Fr should be <0.86 or >1.13 to prevent this.

If water depth is deeper than the critical depth (y_c), then the flow is subcritical.

If the water depth is shallower than the critical depth, then the flow is supercritical.



$$H_0 = y + \frac{V^2}{2g} = y + \frac{q^2}{2gy^2}$$

Figure 7–8 Specific energy diagram.

In the subcritical zone, the water depth component is much larger than the velocity head

In the supercritical zone, the velocity head component is much larger than the water depth.

Prasuhn 1987

State of Flow in Open Channels, as determined by Reynolds and Froude numbers

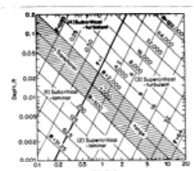


Fig. 1-3. Depth-velocity relationships for four regimes of open-channel flow. (After Robertson and Rouse [30].) (in Ven Te Chow - Open Channel Hydraulics, 1959)

Reynolds number: ratio of inertia to viscosity:

$R = \frac{VL}{\nu}$

V = water velocity (ft/sec)
L = characteristic length (ft) [usually taken as the hydraulic radius, R]
ν = kinematic viscosity of water [1.08x10^{-5} ft^2/s at 70°

if R < 500, then laminar flow
if R > 2,000, then turbulent flow

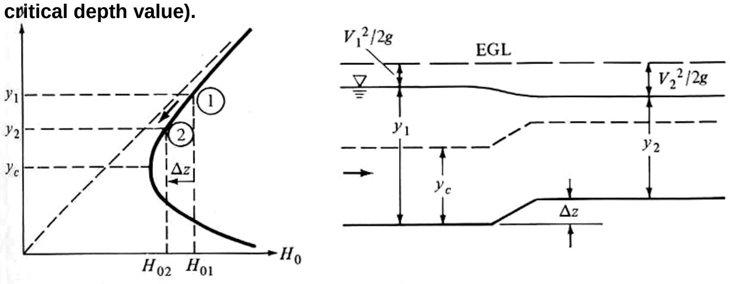
Froude number: ratio of inertial forces to gravity forces

$F = \frac{V}{\sqrt{gL}}$

V = water velocity (ft/sec)
g = acceleration of gravity (32.2 ft/sec^2)
L = characteristic length [usually taken as the hydraulic depth = $\frac{A}{T}$; T = width of free water surface]

if F = 1; then critical flow
if F < 1; then subcritical flow
if F > 1; then supercritical flow

In this example, the upstream flow is subcritical (deeper than the critical depth). When the water approaches an upward step, the specific energy calculation results in two possible water depth solutions. The correct downstream water depth solution is on the same limb of the specific energy diagram as the upstream water depth. In this case, the flow is still subcritical, although the water depth actually decreases (but it cannot pass through the critical depth value).

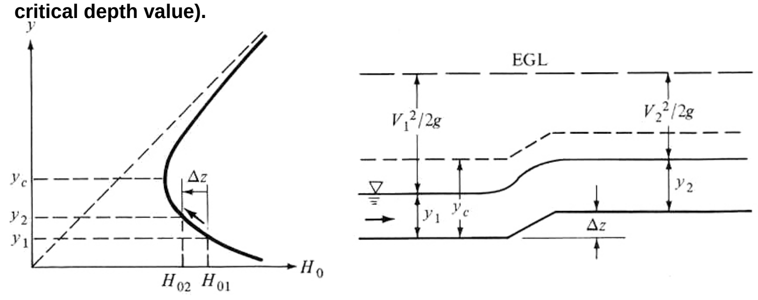


(a) Specific energy diagram (b) Water surface profile

Figure 7–9 Subcritical transition with upward step.

Prasuhn 1987

In this example, the upstream flow is supercritical (shallower than the critical depth). When the water approaches an upward step, the specific energy calculation results in two possible water depth solutions. The correct downstream water depth solution is on the same limb of the specific energy diagram as the upstream water depth. In this case, the flow is still supercritical: the water depth increases (but it cannot pass through the critical depth value).



(a) Specific energy diagram (b) Water surface profile

Figure 7–10 Supercritical transition with upward step.

Prasuhn 1987

In a rectangular channel, the critical depth can be easily calculated using a unit width flow rate:

$$q = \frac{Q}{b}$$

Where b is the width of the rectangular channel

The critical flow depth can then be calculated as:

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

and the minimum specific energy in a rectangular channel is therefore:

$$E_c = \frac{3}{2} y_c$$

Example Problem: Determine the Downstream Water Depth when Affected by Channel Bottom Rise (ex. 7-3, Prasuhn 1987)

Determine the downstream depth in a horizontal rectangular channel in which the bottom rises 0.5 ft, if the steady flow discharge is 300 cfs, the channel width is 12 ft, and the upstream depth is 4 ft.

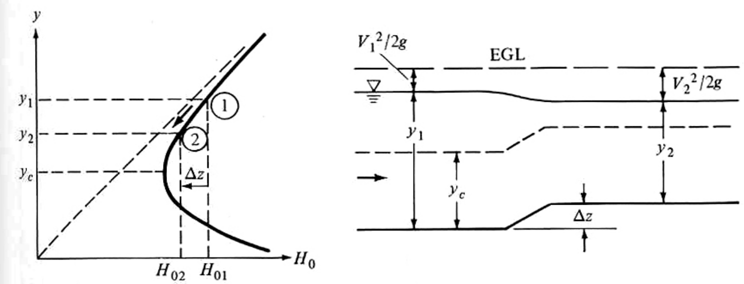
The discharge per unit width is:

$$q = \frac{Q}{b} = \frac{300\, ft^3/\sec}{12\, ft} = 25 cfs/\ ft$$

The critical depth is therefore:

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left(\frac{(25 cfs/\ ft)^2}{32.2\, ft/\sec^2}\right)^{1/3} = 2.69\, ft$$

The upstream depth is therefore subcritical.



(a) Specific energy diagram (b) Water surface profile

Figure 7–9 Subcritical transition with upward step.

The upstream specific energy is calculated to be:

$$H_{01} = y_1 + \frac{q^2}{2gy_1^2} = 4\,ft + \frac{(25cfs/\,ft)^2}{2(32.2\,ft/\sec^2)(4\,ft)^2} = 4.61\,ft$$

and the corresponding downstream specific energy is:

$$H_{02} = H_{01} - \Delta z = y_2 + \frac{q^2}{2gy_2^2}$$

$$H_{02} = 4.61\,ft - 0.5\,ft = 4.11\,ft = y_2 + \frac{(25cfs/\,ft)^2}{2(32.2\,ft/\sec^2)y_2^2}$$

From the subcritical position on the specific energy diagram, the depth and the water surface elevation will both decrease downstream over the "bump" in the channel bottom. Therefore, y_2 must be greater than y_c and less than y_1-Δz:

2.69 ft < y_2 < 3.5 ft

Solving the equation by iteration within this range results in the solution of y_2 = 3.09 ft. The trial solutions can be used to draw in the specific energy diagram.

In-Class Problem

Determine the downstream depth in a horizontal rectangular channel in which the bottom rises 0.75 ft, if the steady flow discharge is 550 cfs, the channel width is 5 ft, and the upstream depth is 6 ft. Also draw the specific energy diagram for this problem.

“Choke”

What happens when Δz is increased to a greater and greater value under subcritical conditions? As Δz increases, H_{o2} must also continue to decrease. Therefore, y_2 decreases as well. The limit is reached for subcritical flow when y_2 equals the critical depth at which point the transition becomes a “choke.” A further increase in Δz results in the impossible situation where H_{o2} is less than H_{omin} (there would be no positive solution to H_{o2}): the upstream flow has insufficient energy to pass through the transition at the specified discharge.

The flow will not cease, but will adjust itself to either a lower discharge or an increase in specific energy. The flow will likely not change due to upstream flow sources. The upstream flow will increase both its upstream depth and specific energy by means of a gentle swell or a series of small waves that travel upstream. The new upstream depth will be such that the flow can just pass the transition and y_2 will equal y_c, and H_{o2} will equal H_{omin}. H_{o1} will exceed H_{omin} by the value of Δz . This transition is called a choke since the critical depth prevails regardless of the increase in upstream energy.

During supercritical flow conditions, the flow behavior is different as Δ z increases. A choke occurs when the minimum specific energy is reached. However, when additional Δ z occurs, a surge (wall of water) moves upstream. When equilibrium is reached, the supercritical flow will have been replaced by the identical subcritical flow discussed above, and the transition will continue to act as a choke.

(summarized from Prasuhn 1987)